MICROLOCAL SINGULARITIES AND SCATTERING THEORY FOR SCHRÖDINGER EQUATIONS ON MANIFOLDS

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We present two recent results on Schrödinger equations on manifolds: (a) the characterization of microlocal singularities of solutions; (b) an FIO (Fourier integral operator) representation of the scattering matrix. These seemingly quite different results are proved by the same set of simple ideas: (1) a construction of semiclassical solutions to the Heisenberg equation; (2) scattering theory for classical mechanics to control the long-time behavior of the semiclassical solution; (3) scattering theory in the polar coordinate. In the proof of corresponding results, we employ different semiclassical parameters, or equivalently, different scalings. For (a), we use scaling in the momentum variable to analyze the microlocal singularities, whereas we use scaling in the radial variable for (b).

These results are joint work with K. Ito (Tsukuba University), and are published in [1, 2, 3, 4].

Keywords: microlocal singularities, scattering theory, semiclassical analysis

[1] K. Ito and S. Nakamura: "Singularities of solutions to Schrödinger equation on scattering manifold". American J. Math. **131** (2009) 1835–1865.

[2] K. Ito and S. Nakamura: "Time-dependent scattering theory for Schrödinger operators on scattering manifolds". J. London Math. Soc. **81** (2010) 774–792.

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[4] K. Ito and S. Nakamura: "Microlocal properties of scattering matrices for Schrödinger equations on scattering manifolds". Preprint 2011 February (`http://arxiv.org/abs/1102.2009`)