UNIVERSALITY FOR β-ENSEMBLES

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Wigner stated the general hypothesis that the distribution of eigenvalue spacings of large complicated quantum systems is universal in the sense that it depends only on the symmetry class of the physical system but not on other detailed structures. The simplest case for this hypothesis concerns large but finite dimensional matrices. Spectacular progress was done in the past two decades to prove universality of random matrices presenting an orthogonal, unitary or symplectic invariance. These models correspond to log-gases with respective inverse temperature 1, 2 or 4. I will report on a joint work with L. Erdős and H.-T. Yau, which yields universality for the log-gases at arbitrary temperature, for analytic external potential, at the microscopic scale. A main step consists in the optimal localization of the particles, and the involved techniques include a multiscale analysis and a local logarithmic Sobolev inequality.

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