APPLICATIONS OF RANDOM MATRICES TO OPERATOR ALGEBRA THEORY

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Since Wigner's pioneering work from 1955 random matrices have been an important tool in Mathematical Physics. After Voiculescu in 1991–95 used random matrices to solve some deep open problems about von Neumann algebras, random matrices have also played a key role in operator algebra theory. In 2005 Steen Thorbjørnsen and the speaker were able to solve an old problem on C^*-algebras, by making careful estimates of the largest and smallest eigenvalues in random ensembles, which can be expressed as (non-commutative) polynomials in two or more independent GUE-random matrices, [1]. Shortly after we obtained (in collaboration with Hanne Schultz) similar estimates for polynomials in GOE- and GSE- matrices [2], but the corresponding problem for polynomials in two or more non-Gaussian random matrices with independent entries was solved only recently by Greg Anderson (2011).

Keywords: Random matrices, von Neumann algebras, C^*-algebras

[1] Haagerup, Uffe; Thorbjørnsen, Steen, A new application of random matrices: $\mathrm{Ext}(C^*_{\mathrm{red}}(F_2))$ is not a group, Ann. of Math. (2) 162 (2005), no. 2, 711–775.

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