INSTABILITY IN NEARLY INTEGRABLE HAMILTONIAN SYSTEMS: GEOMETRIC METHODS.

Amadeu Delshams, Rafael de la LLave, **Tere M. Seara**
Universitat Politecnica de Catalunya, Georgia Tech, Universitat Politecnica de Catalunya

There are many systems that appear in applications that have negligible friction, like the models of celestial mechanics and Astrodynamics, motion of charged particles in magnetic fields chemical reactions, etc.

A general model for this kind of systems is to consider time periodic pertubations of integrable Hamiltonian systems with 2 or more degrees of freedom.

One problem that has attracted attention for a long time since the example of Arnold in 1964 [1] is whether the effect of perturbations accumulate over time and lead to large effects (instability) or whether these effects average out (stability).

In this talk we present some mechanisms that cause instabilitites for general perturbations. We use the so called geometric methods, which work for a priori-unstable systems, where the unperturbed system has some (possibly weakly) hyperbolic object with stable and unstable manifolds.

The main technique is to develop a toolkit to study, in a unified way, tori of different topologies and their invariant manifolds, their intersections as well as shadowing properties of these bi-asymptotic orbits. Part of this toolkit is to unify standard techniques (normally hyperbolic manifolds, KAM theory, averaging theory) so that they can work together. A fundamental tool used here is the scattering map of normally hyperbolic invariant manifolds.

The conditions needed are explicit and are based in the computation of a general Melnikov function. Therefore, they can be checked in specific examples. When the hyperbolic structure of the system is weakly hyperbolic, this Melnikov function is exponentially small as happens in some problems of celestial mechanics, as the restricted three body problem.

Keywords: Normally hyperbolic invariant manifolds, resonances, primary and secondary tori, transition chains, scattering map.

[1] V. Arnold. Instability of dynamical systems with several degrees of freedom. *Sov. Math. Doklady*, 5:581–585, 1964.