IS A RANDOM STATE ENTANGLED ?

Guillaume Aubrun, Stanisław Szarek, Deping Ye
Université de Lyon, Case Western Reserve University and Université Paris 6, Memorial University of Newfoundland

Let ρ be a random mixed state on $\mathcal{H} = \mathbf{C}^d \otimes \mathbf{C}^d$, obtained as the partial trace, over the environment $\mathbf{C}^s$, of a random pure state on $\mathcal{H} \otimes \mathbf{C}^s$ with uniform distribution.

The dichotomy "entanglement vs separability" reflects into a threshold $s_0(d)$ for the environment dimension s, which satisfies the following for any $\varepsilon > 0$ and large enough d:

- if $s \leqslant (1-\varepsilon)s_0(d)$, then ρ is typically entangled,
- if $s \geqslant (1+\varepsilon)s_0(d)$, then ρ is typically separable.

The threshold $s_0(d)$ is defined through geometric properties of the set of separable states. Using results from high-dimensional convex geometry, we obtain the estimation $d^3 \lesssim s_0(d) \lesssim d^3 \log^2 d$. We also compute thresholds for other properties related to separability, such as the PPT criterion.

Keywords: Quantum information theory, random states, entanglement

[1] Physical Review A (Rapid Communications) 85, 030302 (2012)