A FIELD THEORETIC APPROACH TO STOCHASTIC CALCULUS: EXPLORING ROUGH PATHS

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We shall present here a series of recent articles [1, 2] dedicated to the definition of iterated integrals for stochastic processes with very low Hölder regularity index α. According to the general principles of Lyons' theory of *rough paths*, iterated integrals, a priori ill-defined, should either be seen as some limit of actual iterated integrals, or equivalently as a stack of data satisfying some Hölder regularity and algebraic axioms; such data define unambiguously a stochastic calculus and 'rough path solutions' to stochastic differential equations.

For many processes with $\alpha \leq 1/4$, in particular for Gaussian processes (the main example being fractional Brownian motion), a proper definition of iterated integrals was not available with standard tools of stochastic calculus. Through an approach (*Fourier normal ordering*) combining Hopf algebraic combinatorics, multi-scale expansions, Feynman diagram renormalization and finally constructive field theory, we gave a satisfactory answer to this problem. Ultimately the underlying structure is provided by the operator product expansions of 'composite operators' built out of the original process, and should be the key to the probabilistic study of solutions of stochastic differential equations driven by it.

Keywords: rough paths, iterated integrals, renormalization, constructive field theory

[1] J. Unterberger. *Hölder-continuous paths by Fourier normal ordering*, Comm. Math. Phys. **298** (1), 1–36 (2010).

[2] J. Magnen, J. Unterberger. *From constructive theory to fractional stochastic calculus. (I) An introduction: rough path theory and perturbative heuristics. (II) Constructive proof of convergence for the Lévy area of fractional Brownian motion with Hurst index* $\alpha \in (1/8, 1/4)$, Ann. Henri Poincaré **12**, 1199-1226 and **13** (2), 209–270 (2011).