CRITICALITY WITHOUT FRUSTRATION FOR QUANTUM SPIN-1 CHAINS

Sergey Bravyi
IBM T.J. Watson Research Center

Frustration-free (FF) spin chains have a property that their ground state minimizes all individual terms in the chain Hamiltonian. We ask how entangled can the ground state of a FF quantum spin-s chain with nearest-neighbor interactions be for small values of s. While FF spin-1/2 chains are known to have unentangled ground states, the case $s = 1$ remains less explored. We propose the first example of a FF translation-invariant spin-1 chain that has a unique highly entangled ground state and exhibits some signatures of a critical behavior. The ground state can be viewed as the uniform superposition of balanced strings of left and right parentheses separated by empty spaces. Entanglement entropy of one half of the chain scales as $\frac{1}{2}\log(n)+O(1)$, where n is the number of spins. We prove that the energy gap above the ground state is polynomial in $1/n$. The proof relies on a new result concerning statistics of Dyck paths which might be of independent interest.

Keywords: quantum phase transitions, entanglement area law, Dyck paths

[1] S. Bravyi, L. Caha, R. Movassagh, D. Nagaj, and P. Shor, arXiv:1203.5801.