UNSTABLE MANIFOLDS AND L^2 NONLINEAR INSTABILITY OF EULER EQUATIONS

Zhiwu Lin, **Chongchun Zeng**
Georgia Institute of Technology, Georgia Institute of Technology

We consider the nonlinear instability of a steady state v_0 of the Euler equation in a fixed bounded domain in $\mathbf{R}^n$. When considered in H^s, $s > 1$, at the linear level, the stretching of the steady fluid trajectories induces unstable essential spectrum which corresponds to linear instability at small spatial scales and the corresponding growth rate depends on the choice of the space H^s. Therefore, more physically interesting linear instability relies on the unstable eigenvalues which correspond to large spatial scales. In the case when the linearized Euler equation at v_0 has an exponential dichotomy of unstable (from eigenvalues) and center-stable directions, most of the previous results obtaining the expected nonlinear instability in L^2 (the energy space) were based on the vorticity formulation and therefore only work in 2-dim. In this talk, we prove, in any dimensions, the existence of the unique local unstable manifold of v_0, under certain conditions, and thus its nonlinear instability. Our approach is based on the observation that the Euler equation on a fixed domain is an ODE on an infinite dimensional manifold of volume preserving maps in in function spaces.

Keywords: Euler equation, nonlinear instability, unstable manifolds.