

# ELLIPTIC HYPERGEOMETRY OF SUPERSYMMETRIC DUALITIES

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Recently there was a substantial progress in understanding of supersymmetric theories (in particular, their BPS spectrum) in space-times of different dimensions due to the exact computation of superconformal indices and partition functions. It resulted in a better understanding of electro-magnetic (or the strong-weak) dualities. Consistent consideration of such dualities has been given in the maximally extended  $\mathcal{N} = 4$ ,  $\mathcal{N} = 2$  and  $\mathcal{N} = 1$  theories in four dimensions and partially in three dimensions. There exists a whole zoo of different Seiberg dualities for  $4d$   $\mathcal{N} = 1$  SYM theories and their  $3d$   $\mathcal{N} = 2$  analogues, and their classification using some group-theoretical approach is still open.

Highly nontrivial generalizations of the Witten index were proposed by Römelsberger and Kinney et al. These superconformal indices describe the structure of BPS states protected by one supercharge and its conjugate which cannot be combined to form long multiplets. It appeared that equalities of superconformal indices are equivalent to the exact computability of elliptic beta integrals discovered earlier by V. Spiridonov or nontrivial symmetry transformations for higher order elliptic hypergeometric functions on root systems.

We systematically investigate this approach to supersymmetric dualities using the theory of elliptic hypergeometric integrals. In a series of papers [1, 2] devoted to this subject a full list of known  $\mathcal{N} = 1$  supersymmetric quantum field theories related by the Seiberg duality conjectures for  $SU(N)$ ,  $SP(2N)$ ,  $SO(N)$  and  $G_2$  gauge groups. Many of these dualities are new, not considered earlier in the literature. For all these theories the superconformal indices are constructed. Equalities of indices in dual field theories lead to various identities for the elliptic hypergeometric integrals. About half of them was proven earlier, but another half represents new challenging conjectures which desire rigorous mathematical proof. A connection to knot theory and vortex partition function of two-dimensional supersymmetric theories is discussed.

*Keywords:* Supersymmetric Dualities, Superconformal Index, Elliptic Hypergeometric Integrals.

- [1] V. P. Spiridonov and G. S. Vartanov, *Elliptic Hypergeometry of Supersymmetric Dualities*, Commun. Math. Phys. **304** (2011) 797, arXiv:0910.5944 [hep-th].
- [2] V. P. Spiridonov and G. S. Vartanov, *Elliptic hypergeometry of supersymmetric dualities II. Orthogonal groups, knots, and vortices*, arXiv:1107.5788 [hep-th].