REFINED SEMI-CLASSICAL ASYMPTOTICS FOR THE LAPLACIAN AND THE FRACTIONAL LAPLACIAN

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We consider the Laplacian $-\Delta$ and the fractional Laplacian $(-\Delta)^s$, $0 < s < 1$, on a domain and investigate the asymptotic behavior of the eigenvalues. Extending methods from semi-classical analysis we prove a two-term formula for the sum of the eigenvalues with the leading (Weyl) term given by the volume and the subleading term depending on properties of the boundary. These results are valid under very weak assumptions on the regularity of the boundary. It is remarkable that, despite the fact that the fractional Laplacian is a non-local operator, even the second term has a local form depending only on the surface area of the boundary. In the local case, $s = 1$, we show how the second term depends on different boundary conditions, including Dirichlet, Neumann and varying Robin conditions. This is joint work with Rupert L. Frank.