RENORMALIZATION AND RIGIDITY THEORY FOR CIRCLE MAPS WITH BREAKS

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Rigidity theory of circle diffeomorphisms, which concerns C^1 -smooth conjugacy to a rigid rotation, is a classic problem in dynamical systems that was initiated by Arnol'd and settled by Herman. We present the full renormalization and rigidity theory for circle maps with breaks, i.e., circle diffeomorphisms with a single singular point where the derivative has a jump discontinuity. We prove that renormalizations of any two $C^{2+\alpha}$ -smooth circle maps with breaks, with the same irrational rotation number and the same size of the break, approach each other exponentially fast. As a corollary, we obtain a strong rigidity statement for such maps: for almost all irrational rotation numbers, any two circle maps with breaks, with the same rotation number and the same size of the break, are C^1 -smoothly conjugate to each other. Finally, we prove that the latter result cannot be extended to all irrational rotation numbers.