HYDRODYNAMIC LIMIT FOR HARMONIC CRYSTALS

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The talk is devoted to the problem of deriving hydrodynamic equations from the microscopic Hamilton dynamics. As the model we consider the many-dimensional crystal lattice in the harmonic approximation. The initial data are s upposed to be a random function. We introduce the family of initial measures depending on a small scaling parameter ε and assume that the initial measures are locally homogeneous for space translations of order much less than $1/\varepsilon$ and nonhomogeneous for translations of order $1/\varepsilon$. Given nonzero $t \in \mathbb{R}$, and $x \in \mathbb{R}^d$, we consider the distributions of random solution in the time moments t/ε^k $(k = 1, 2, ...)$ and at lattice points close to $[x/\varepsilon]$. The main goal is to study the asymptotics of these distributions as $\varepsilon \to 0$ and to derive the limiting equations which can be interpreted as the Euler and Navier-Stokes equations for our model. In one-dimensional case, the similar results were obtained by Dobrushin et al. for the infinite chain of harmonic oscillators. For more details, see [T.V. Dudnikova, Caricature of hydrodynamics for lattice dynamics, preprint at Arxiv:1110.0616].