# QUANTUM STEIN'S LEMMA, ENTROPY INEQUALITIES AND LIEB'S CONCAVITY THEOREM

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We derive the monotonicity of the quantum relative entropy by an elementary operational argument based on Stein's lemma in quantum hypothesis testing. For the latter we present an elementary and short proof that requires the law of large numbers only. Joint convexity of the quantum relative entropy is proven too, resulting in a self-contained elementary version of Tropp's approach to Lieb's concavity theorem, according to which the map $\operatorname{tr}(\exp(h + \log a))$ is concave in $a$ on positive operators for self-adjoint $h$.