

ON THE TROTTER-KATO PRODUCT FORMULA FOR
UNITARY GROUPS

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Recently [1] we obtained some new results on the Trotter-Kato product formula for unitary groups. Let A and B be non-negative self-adjoint operators in a separable Hilbert space \mathcal{H} such that their form-sum C is densely defined. It is shown that the Trotter product formula holds for imaginary parameter values in the L^2 -norm, that is, one has

$$\lim_{n \rightarrow \infty} \int_{-T}^T \left\| \left(e^{-itA/n} e^{-itB/n} \right)^n h - e^{-itC} h \right\|^2 dt = 0$$

for each element $h \in \mathcal{H}$ and any $T > 0$. This result is extended to the class of holomorphic Kato functions, to which the exponential function belongs. Moreover, for a class of so-called *admissible functions*: $\phi(\cdot), \psi(\cdot) : \mathbb{R}_+ \rightarrow \mathbb{C}$, satisfying some additional conditions we proved that

$$s - \lim_{n \rightarrow \infty} (\phi(tA/n)\psi(tB/n))^n = e^{-itC}$$

is true uniformly on $[0, T] \ni t$ for any $T > 0$.

This communication is a result of a common project with **Pavel Exner** (Doppler Institute, Prague) and **Hagen Neidhardt** (Weierstrass Institute, Berlin).

- [1] P. Exner, H. Neidhardt, V. Zagrebnov: Remarks on the Trotter-Kato Product Formula for Unitary Groups, *Integ.Equ.Oper.Theory* **69**, 451-478 (2011)