AN UPPER BOUND ON THE GROUND STATE ENERGY OF A 4-DIMENSIONAL DILUTE BOSE GAS

Anders Aaen
Department of Mathematics, Aarhus University

We consider a low density Bose gas in $n > 3$ spatial dimensions. In dimension $n = 3$, the leading order term of the ground state energy per particle in the thermodynamic limit is $4\pi a\rho$, where a is the scattering length of the two-body interaction and ρ is the density of the gas. We show that an analogous result holds in any dimension $n > 3$. Moreover, we consider a 4-dimensional Bose gas, with a scaled two-body interaction, and calculate a rigorous second order upper bound on the ground state energy per particle in the thermodynamic limit. This is inspired by a recent paper of Yang, where the pseudo-potential method, used to predict the Lee-Huang-Yang formula, is reexamined in dimension 2, 4 and 5. Up to a small constant, depending on the scaling of the potential, our upper bound agrees with the prediction of Yang. A similar calculation for a 3-dimensional Bose gas was recently carried out by Erdős-Schlein-Yau. However, their method has to be modified in the 4-dimensional case due to an ultraviolet divergence of a certain integral.