DIFFUSION FOR A QUANTUM PARTICLE COUPLED TO A PHONON GAS

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I report on work [1, 2] whose aim is to display diffusion in a many-body system described by Hamiltonian dynamics. More concretely, our model consists of a particle weakly coupled to a gas of free phonons. Initially the phonon gas is in a thermal state and the particle sits at the origin. For intermediate times, the motion of the particle is well-described by a Markovian Boltzmann-type equation, corresponding to the simplest 'interaction events' between particle and phonons. This Boltzmann-type equation defines a diffusion constant D_0. For long times, one has to take into account more complicated interactions, corresponding to 'recollisions' in a classical setup. We prove that the motion of the particle is diffusive, i.e. the central limit theorem holds for the variable $X_t/\sqrt{t}$, where X_t is the position of the particle and t is time. The recollisions renormalize the diffusion constant $D_0 \to D$, and the control of this phenomenon is the main technical point of our analysis.

Keywords: quantum field theory, irreversibility, diffusion

[1] W. De Roeck and J. Fröhlich. Diffusion of a massive quantum particle coupled to a quasi-free thermal medium. *Communications in Mathematical Physics*, **303**:613–707, 2011. 10.1007/s00220-011-1222-0.

[2] W. De Roeck and A. Kupiainen. Diffusion for a quantum particle coupled to phonons in $d \geq 3$. *submitted to Communications in Mathematical Physics.* arXiv:1107.4832.