SOLVING THE KPZ EQUATION

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The KPZ equation is the stochastic PDE formally given by

$$\partial_t h = \partial_x^2 h + (\partial_x h)^2 + \xi \,, \tag{1}$$

where ξ denotes space-time white noise. It was originally introduced in the eighties as a model of surface growth, but it was soon realised that its solution is a much more universal object describing the crossover between the Gaussian universality class and the KPZ universality class. The mathematical proof of its universality however is still an open problem, in particular because of the lack of a good approximation theory for the equation. Indeed, the only known way so far to mathematically interpret solutions to the KPZ equation is to reduce it to a linear stochastic PDE via a non-linear transformation called the Cole-Hopf transform. Unfortunately, the resulting linear equation does itself lack a good approximation theory and many microscopic models do not behave well under the Cole-Hopf transform.

In this talk, we present a new notion of solution to the KPZ equation that bypasses the use of the Cole-Hopf transform. Our approach also allows to factorise the solution map into a "universal" (i.e. independent of initial condition) measurable map, composed with a solution map with good continuity properties. This lays the foundations for a robust approximation theory to the KPZ equation, which is needed to prove its universality. As a byproduct of the construction, we obtain very detailed regularity estimates on the solutions, as well as a new homogenisation result.