2D COULOMB GAS, ABRIKOSOV LATTICE AND RENORMALIZED ENERGY

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In joint work with Etienne Sandier, we studied the statistical mechanics of a classical two-dimensional Coulomb gas, particular cases of which also correspond to random matrix ensembles.

We connect the problem to the "renormalized energy" W, a Coulombian interaction for an infinite set of points in the plane that we introduced in connection to the Ginzburg-Landau model, and whose minimum is expected to be achieved by the "Abrikosov" triangular lattice.

I will briefly allude to the results obtained on Ginzburg-Landau and focus mostly on the Coulomb gas system. Results include a next order asymptotic expansion of the partition function, and various characterizations of the behavior of the system at the microscopic scale. When the temperature tends to zero (the limit also corresponds to "weighted Fekete sets") we show that the system tends to "crystallize" to a minimizer of W.

Keywords: Coulomb gas, Ginzburg-Landau model, Abrikosov lattice, crystallization, random matrices, Ginibre ensemble.

[1] E. Sandier, S. Serfaty, From the Ginzburg-Landau Model to Vortex Lattice Problems, *Comm. Math. Phys.*, 2012, available online.

[2] E. Sandier, S. Serfaty, 2D Coulomb Gases and the Renormalized Energy, `http://arxiv.org/abs/1201.3503v1`