ANDERSON ORTHOGONALITY CATASTROPHE

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We compute the asymptotics of the so-called Anderson integral $I_N := \mathrm{tr} P_N(\mathbf{1} - \Pi_N)$. Here, P_N and Π_N are the spectral projections of the Schrödinger operators H_0 and $H := H_0 + V$, respectively, corresponding to the first N eigenvalues. The operators H_0 and H are defined in $L^2[-L, L]$ with Dirichlet boundary conditions. We show that $I_N \sim \gamma \ln N$ in the thermodynamic limit, i.e $N, L \to \infty$ with fixed particle density N/L. The constant γ depends on the potential V. The proof uses Riesz's integral formula for spectral projections and Krein's resolvent formula. Through the latter appears the square of the unperturbed resolvent, which converges to a Dirac delta function and thus singles out the asymptotics. Anderson used I_N in 1967 to study the ground state transition probability of a system of N free fermions that is exposed to a sudden perturbation.